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NUMERICAL MODELS FOR FLOOD WAVE PROPAGATION IN MOUNTAIN RIVERS: AN APPLICATION TO THE TORRENT MALLERO (ITALY)

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INTRODUCTION

In contrast to the large lowland rivers, mountain streams are generally part of a dense hydrographic network, with extremely variable, both in time and space, hydrological, morphological and sedimentological characteristics. On the other hand, even excluding the farthest and steepest branches of the network, mountain streams always have relatively large slopes: as a consequence, not only flood wave propagation is very fast, but the flow in a given reach is basically controlled only by the characteristics of the reach itself (no backwater effects). In mountain rivers, the combination of relatively small water depth with large grain-sizes of bed material, leads to large scale-roughness. Moreover, although the flow pattern at small space scale is highly non-uniform, i.e., the flow locally alternates into subcritical and supercritical states (transcritical flow), in a relatively long reach, let say of the order of magnitude of the bottom width , the average flow condition is quite well represented by a quasi uniform flow. This generally allows, from one side, to represent each reach by its averaged geometric characteristic and global roughness parameters, and on the other hand, to apply a simplified description of the fluid motion assuming uniform flow for each reach. Obviously, as mentioned before, this simplification is valid only if the mean friction slope is not much different from the mean bottom slope, that is when backwater effects do not exists.

In this work an attempt is made to testing some numerical models based on both complete and simplified one-dimensional unsteady open channel flow equations. The numerical models were applied to the exceptional event that took place in July of 1987 in the torrent Mallero (Valtellina- Northern Italy). A dynamic, diffusive and kinematic mathematical models based on Mac Cormack's scheme, as well as a kinematic wave model based on FTBS (predictor) - Four points (corrector) numerical scheme were used.

MATHEMATICAL MODELS

Full hydrodynamic model

The unsteady flow equations in conservation law form for irregular cross sections are expressed as follow (Bellos 1995):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left[\frac{Q^2}{A} + \frac{F_h}{\rho}\right] = gA(S_0 - S_f) + q_l u_l + \frac{F_{lx}}{\rho} \tag{2}$$

in which, A: cross section area, Q: flow discharge, u: mean flow velocity, q_l: lateral inflow for unit length, F_h: hydrostatic pressure force in the cross section, F_{lx}: longitudinal pressure force, u_l: velocity component of the lateral inflow in the mean stream direction, g: gravity acceleration, ρ: water density and S_0, S_f : bed and friction slopes respectively. All units are in the SI system.

Diffusive model

In mountain rivers the gravitational forces are dominant respect to the inertial ones, thus equation (2) can be written in the following way:

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$$A\frac{\partial u}{\partial t}+u\frac{\partial A}{\partial t}+u^2\frac{\partial A}{\partial x}+2Au\frac{\partial u}{\partial x}+\frac{\partial(F_h/\rho)}{\partial x}=gA(S_0-S_f)+q_l u_l+\frac{F_{lx}}{\rho} \qquad (3)$$

substituting eqn. (1) in (3) and assuming that the inertial terms $\partial u/\partial t+u\partial u/\partial x$ are negligible, eqn. (2) takes the form:

$$\frac{\partial(F_h/\rho)}{\partial x}=gA(S_0-S_f)+q_l(u_l-u)+\frac{F_{lx}}{\rho} \qquad (4)$$

Eqns. (1) and (4) constitutes the diffusive model. In this model eqn. (4) is time independent and can be written in the form:

$$S_f=S_0-\frac{\dfrac{\partial(F_h/\rho)}{\partial x}-\dfrac{F_{lx}}{\rho}-q_l(u_l-u)}{gA} \qquad (5)$$

where S_f is expressed by the Manning's equation:

$$Q=\frac{1}{n}AR^{2/3}S_f^{1/2} \qquad (6)$$

Thus, discretizing the derivative $\frac{\partial}{\partial x}(F_h/\rho)$, we have a relationship between Q and the geometrical characteristics of the cross sectional area of the river. Finally, eqn. (1) assume the form:

$$\frac{\partial A}{\partial t}+\frac{\partial Q(A)}{\partial x}=q_l \qquad (7)$$

If the cross sections are rectangular and prismatic without lateral inflow, eqn. (5) can be written as:

$$\frac{\partial h}{\partial x}=S_0-S_f \qquad (8)$$

in which h is the water depth. This form is known in the literature as diffusive wave model.

Kinematic model

If the pressure differential term can be neglected, then, the derivative $\frac{\partial}{\partial x}(F_h/\rho)$ is set equal to zero and as a consequence eqn. (5) assume the form:

$$S_f=S_0-\frac{F_{lx}/\rho-q_l(u_l-u)}{gA} \qquad (9)$$

Eqns. (9) and (1) constitutes the kinematic wave model. In the case of prismatic sections without lateral inflow eqn. (9) assume the well known form:

$$S_0=S_f \qquad (10)$$

Moreover, recalling the unsteady continuity equation, eqn. (1), and noting that the kinematic wave celerity, c, is equal (Cunge et al., 1980; Miller, 1984) to:

$$c=\left.\frac{\partial Q}{\partial A}\right|_{x_o} \qquad (11)$$

and that the time derivative of wetted area A can be written as:

$$\frac{\partial A}{\partial t}=\frac{\partial A}{\partial Q}\frac{\partial Q}{\partial t} \qquad (12)$$

using expression (11) and substituting (12) in (1) we obtain what is often called the kinematic wave equation:

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = cq_l \tag{13}$$

The value of c depends upon the equation used to represent the resistance to flow, for example, using eqn. (6) we have: $c=(5/3)\,u$ or using Chezy's formula we obtain: $c=(3/2)\,u$.

NUMERICAL MODELS

Full hydrodynamic model

In this case the model is based on the well known Mac Cormack's numerical scheme. In order to smearing the discontinuities caused from the transcritical flow conditions and cross section irregularities, a diffusive term ω was added. The numerical scheme used is an explicit, two steps, second order of accuracy, which reads:

- Predictor:

$$\hat{A}_j = A_j^n - \lambda(Q_{j+1}^n - Q_j^n) + \Delta t q_{lj}^n \tag{14}$$

$$\hat{Q}_j = Q_j^n - \lambda(F_{j+1}^n - F_j^n) + \Delta t D_{lj}^n \tag{15}$$

where, $\lambda = \dfrac{\Delta t}{\Delta x}$, $F = \dfrac{Q}{A} + \dfrac{F_h}{\rho}$, $D = gA(S_0 - S_f) + u_l q_l + \dfrac{F_{lx}}{\rho}$

- Corrector:

$$A_j^{n+1} = \frac{1}{2}\left[\omega A_j^n + \frac{1}{2}(\omega - 1)(A_{j-1}^n - A_{j+1}^n) + \hat{A}_j - \lambda(\hat{Q}_j - \hat{Q}_{j-1}) + \Delta t q_{lj}\right] \tag{16}$$

$$Q_j^{n+1} = \frac{1}{2}\left[\omega Q_j^n + \frac{1}{2}(\omega - 1)(Q_{j-1}^n - Q_{j+1}^n) + \hat{Q}_j - \lambda(\hat{F}_j - \hat{F}_{j-1}) + \Delta t D_j\right] \tag{17}$$

The initial conditions are represented by $Q(x,0)$=constant and $A(x,0)$=calculated with Manning's equation. At the upstream boundary the following conditions are especified $Q(0,t)$=hydrograph (data) and $A(0,t)$=calculated with Manning's equation. At the downstream boundary $Q(L,t)=Q(L-\Delta x,t)$ and $A(L,t)=A(L-\Delta x,t)$ for supercritical flow.

Diffusive model

In this case eqn. (7) must be solved. The numerical scheme, based on Mac Cormack's scheme is:

- Predictor:

$$\hat{A}_j = A_j^n - \lambda(Q_{j+1}^n - Q_j^n) + \Delta t q_{lj}^n \tag{18}$$

from eqns. (5) and (6): $\hat{Q}_j = \hat{Q}(\hat{A}_j)$

- Corrector:

$$A_j^{n+1} = \frac{1}{2}\left[\omega A_j^n + \frac{1}{2}(\omega - 1)(A_{j-1}^n - A_{j+1}^n) + \hat{A}_j - \lambda(\hat{Q}_j - \hat{Q}_{j-1}) + \Delta t \hat{D}_j\right] \tag{19}$$

Initial and boundary conditions are prescribed as previously mentioned for the full hydrodinamic model.

Kinematic model

Mac Cormack's scheme

In this case, as in the above mentioned case, the two steps technique assume the form:

- Predictor:

$$\hat{A}_j = A_j^n - \lambda(Q_{j+1}^n - Q_j^n) + \Delta t q_{lj}^n \tag{20}$$

from eqns. (9) and (10): $\hat{Q}_j = \hat{Q}(\hat{A}_j)$

- Corrector:

$$A_j^{n+1} = \frac{1}{2}\left[\omega A_j^n + \frac{1}{2}(\omega - 1)(A_{j-1}^n - A_{j+1}^n) + \hat{A}_j - \lambda(\hat{Q}_j - \hat{Q}_{j-1}) + \Delta t \hat{D}_j\right] \tag{21}$$

Initial conditions as previously mentioned for the above cases and boundary conditions only specified at the upstream boundary.

FTBS-Four Points scheme

In this case eqn. (13) was solved numerically using a finite difference approximation. A predictor-corrector method was used. The predictor step was performed with a FTBS (Forward Time Backward Space) scheme, while the corrector step was carried out with a Four-Points scheme (Priessmann).

- Predictor:

The time and space derivatives of the water discharge are approximated in the following way:

$$\frac{\partial Q}{\partial t} = \frac{Q_j^\xi - Q_j^n}{\Delta t} \quad , \quad \frac{\partial Q}{\partial x} = \frac{Q_j^n - Q_{j-1}^n}{\Delta x} \tag{22a, 22b}$$

the celerity c is discretized as follows: $c = 0.5\left(c_j^n + c_{j-1}^n\right)$ and the lateral input discharge per unit length is:

$$q_{lj}^n = Q_{lj}^n \big/ \Delta x \tag{23}$$

introducing (22a), (22b) and (23) in (13) and expliciting the predicted discharge at grid point j one gets:

$$Q_j^\xi = Q_j^n - c\frac{\Delta t}{\Delta x}(Q_j^n - Q_{j-1}^n - Q_{lj}^n) \tag{24}$$

- Corrector:

In the corrector step the time and space derivatives of the water discharge are approximated in the following manner:

$$\frac{\partial Q}{\partial t} = \frac{\psi\left(Q_j^{n+1} - Q_j^n\right) + (1-\psi)\left(Q_{j-1}^{n+1} - Q_{j-1}^n\right)}{\Delta t} \tag{25}$$

$$\frac{\partial Q}{\partial x} = \frac{\theta\left(Q_j^{n+1} - Q_{j-1}^{n+1}\right) + (1-\theta)\left(Q_j^n - Q_{j-1}^n\right)}{\Delta x} \tag{26}$$

celerity c, weighted in space and time, is expressed as follows:

$$c = \theta\left[\psi c_j^\xi + (1-\psi)c_{j-1}^\xi\right] + (1-\theta)\left[\psi c_j^n + (1-\psi)c_{j-1}^n\right] \tag{27}$$

substituting (25) and (26) in (13) and rearranging we obtain:

$$\alpha Q_j^{n+1} + \beta Q_j^n + \delta Q_{j-1}^{n+1} - \gamma Q_{j-1}^n - \eta Q_{lj}^{n+1} = 0 \tag{28}$$

in which α, β, δ, γ, and η are coefficients given by:

$$\alpha = \left[\frac{\psi}{\Delta t} + \frac{c\theta}{\Delta x}\right],\ \beta = \left[\frac{c(1-\theta)}{\Delta x} - \frac{\psi}{\Delta t}\right],\ \delta = \left[\frac{(1-\psi)}{\Delta t} - \frac{c\theta}{\Delta x}\right],\ \gamma = \left[\frac{(1-\psi)}{\Delta t} - \frac{c(1-\theta)}{\Delta x}\right],\ \eta = \left[\frac{c}{\Delta x}\right]$$

from eqn. (28) we can obtain the discharge in grid point j at time level n+1 as:

$$Q_j^{n+1} = \frac{\gamma Q_{j-1}^n + \eta Q_{lj}^{n+1} - \beta Q_j^n - \delta Q_{j-1}^{n+1}}{\alpha} \tag{29}$$

The solution $Q(x,t)$ of eqn. (13) in the space domain $0<x<L$ requires one initial value $Q(x,0)$ at each computational point of the modelled domain and one boundary condition $Q(0,t)$. The kinematic wave equation cannot represent backwater effects.

RESISTANCE TO FLOW

As far as the resistance to flow is concerned, in all the models, Manning's coefficient related to grain roughness is calculated by means of the following expression:

$$n = d_{90}^{1/6} / 26 \tag{30}$$

where d_{90} expressed in m is the grain size of bed material for which 90% is finer. However, total flow resistance factors must include free surface instabilities, secondary flows, non-uniform shear stress distribution, cross section irregularities, channel shape, obstructions, vegetation, channel meandering, suspended and bed load (Jarret, 1984). Due to large bed roughness and subsequently great energy dissipation, Mussetter (1989) observed the flow resistance at steeper slopes to be much greater than for flatter slopes. In fact, Jarret (1988) based on measurements performed on 21 mountain rivers in Colorado (USA) proposes the following expression for Manning coefficient:

$$n = 0.32\ S_o^{\ 0.3}\ R^{-0.16} \tag{31}$$

with 0.002 < S_o < 0.052 and 0.15 m < R < 2.2 m, where R is the hydraulic radius. Equation (31), when applied to some reaches of the Mallero torrent (see applications bellow), gives values of n approximately 3-4 times greater than those calculated with equation (30). It was also confirmed by the values of n necessary to simulate adequately flood wave propagation in the Frodolfo torrent (Valfurva-Italy). Then, in the applications, the values of n obtained by applying equation (30) were multiplied by a coefficient equal to 4.

STABILITY CONDITIONS

In all the models the choice of the time step size Δt is subject to Courant-Friedrichs-Lewy (CFL) stability constraint:

$$\Delta t = \sigma \frac{\Delta x}{c_{max}} \tag{32}$$

where 0 < σ < 1 and c_{max} is the maximum value of celerity at a relevant time step. The constant σ is generally set to values close to unity.

In addition to CFL criterion, the time step size in the dynamic model must satisfy the following condition (Terzidis and Strelkoff, 1970; Becker and Yeh, 1972):

$$\Delta t < R^{4/3} / g\, n^2 \left|u\right| \tag{33}$$

In the kinematic model based on FTBS (predictor)-Four Points (corrector) scheme σ can arrive up to 10 without any noticeable instability, numerical diffusion or phase error. In the applications of diffusive and kinematic models based on Mac Cormack's scheme the value of σ was set equal to 0.8, while in the dynamic model it was equal to 0.1.

APPLICATIONS

The above mentioned numerical models were applied to simulate the flood wave propagation in the Mallero torrent (Valtellina-Northern Italy) during the extreme flood event of July 1987.

General description of the basin

The Mallero torrent is a tributary of the Adda river which is the main stream of Valtellina, in the central Alpine region of Northern Italy. The Mallero torrent is 24 km long, starting at the elevation of 1636 m (m.s.l.) from the confluence of the Vazzeda and the Ventina torrents, and ending at the elevation of 282 m on its confluence with the Adda river. The basin of the torrent Mallero and the schematization of the models are shown in Figure 1. The surface of its basin is approximately 319 km^2. The Valtellina region has always been, as has been known since the Middle Age as a place where severe storms accompanied by landslides and overaggradation have occurred. On July 1987 an exceptional event, with a discharge of around 200 years return period, has produced a huge inundation in Valtellina.

Models implementation

The necessary data for the implementation of the models are: longitudinal profile and cross sections of the main stream, granulometric composition of the bottom material, hydrographs corresponding to each tributary as well as hydrograph at the upstream boundary and stage-discharge relations at the downstream boundary.

The available morphological and sedimentological data obtained from 61 different cross sections (see Table 1) along the entire reach of 24 km were processed and bed slopes, d_{90} of bed material, and geometric characteristics of cross sections were determined and incorporated in the models. In order to generate the required data in each computational point of the modelled reach specially developed interpolation subroutines were used.

Tributaries were not included in the model, except as a lateral input of water. The hydrographs at the downstream end of each tributary were computed by applying a rainfall-runoff model (Di Silvio and Peviani, 1989). The hydrographs corresponding to each tributary are presented in Figures 2 and 3.

In the models based on Mac Cormack's scheme the diffusive coefficient ω was equal to 0.5, while in the model based on FTBS-Four Points scheme the values of θ and ψ were equal to 0.6 and 0.5 respectively.

For all the simplified models the space step size was set equal to Δx=250 m, while in the dynamic model it was set equal to Δx=50m.

In Figures 4, 5 and 6 the calculated hydrographs corresponding to distances +9 km, +18 km and +24 km respectively are presented. From these Figures it is observed that the flow hydrographs calculated with the kinematic and the diffusive models based on Mac Cormack's scheme are almost identical to those calculated with the kinematic model based on FTBS-Four Points scheme.

In the dynamic model, as was expectable, a more attenuation of the peak discharge occurs. Parasitic oscillations, due to dispersive numerical errors, appears in the initial time steps of the calculation.

CONCLUSIONS

The aim of the present work was to test, in a real case, some numerical models based on both complete and simplified one-dimensional unsteady shallow water equations. The numerical models were applied to simulate the flood wave propagation in the torrent Mallero (Valtellina-Northern Italy) during the exceptional event of July 1987. The following conclusions can be drawn:

- The kinematic models based on Mac Cormack and FTBS-Four Points schemes gives almost identical results. It means that the different numerical schemes do not influence the flood wave propagation characteristics.

- The calculated hydrographs by means of both kinematic (Mac Cormack and FTBS-Four Points) and diffusive (Mac Cormack) models are almost identical. It means that in this case the differential pressure term does not affect so much the final results.

- The dynamic model requires high computational time because of the small Courant number (σ=0.1) necessary to guarantee stability of the model. As was expectable, in the Dynamic model the attenuation of the peak discharge was greater than that obtained with the others models. However, the differences are not significant.

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Figure 1.- Basin of the torrent Mallero and model schematization.

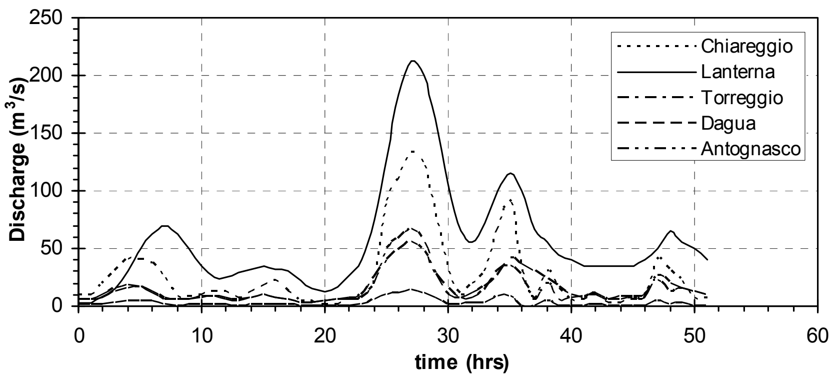

Figure 2.- Hydrographs corresponding to the tributaries of the torrent Mallero from torrents Chiareggio to Antognasco.

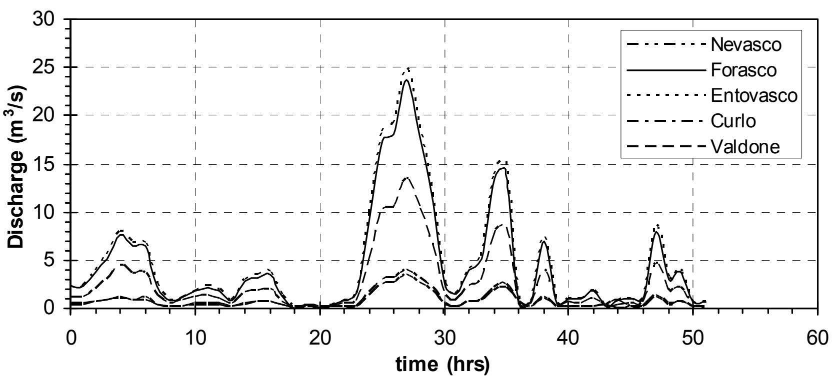

Figure 3.- Hydrographs corresponding to the tributaries of the torrent Mallero, from torrents Nevasco to Valdone.

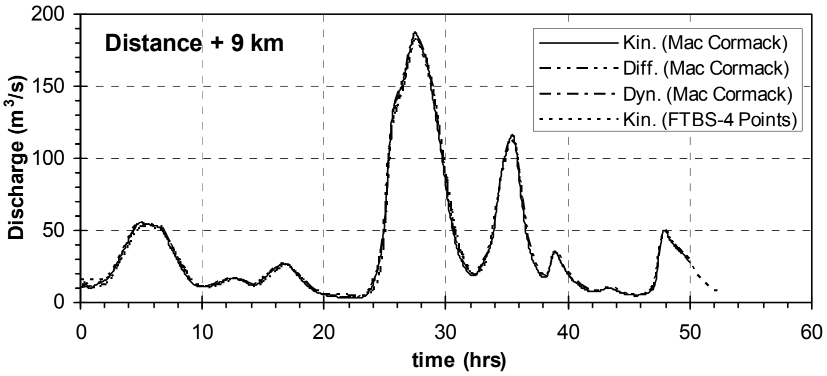

Figure 4.- Calculated hydrographs at the progressive +9 km.

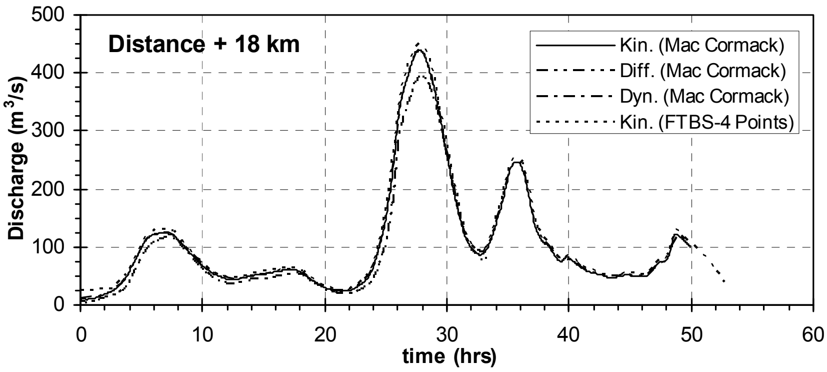

Figure 5.- Calculated hydrographs at the progressive +18 km.

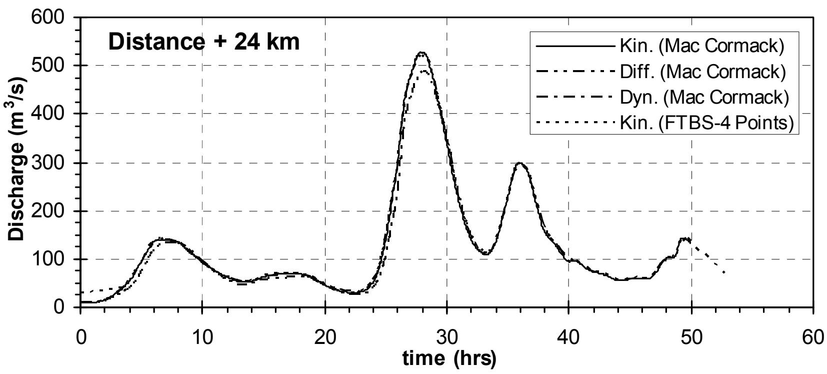

Figure 6.- Calculated hydrographs at the progressive +24 km.

Table 1.- Morphological and sedimentological data.

Pr (m)	B (m)	So	$\beta 1$	$\beta 2$	$\beta 3$	$\beta 4$	Pr (m)	B (m)	So	$\beta 1$	$\beta 2$	$\beta 3$	$\beta 4$
0	12.0	0.1	0.37	0.21	0.26	0.16	14407	20.5	0.062	0.1	0.1	0.34	0.46
1000	15.0	0.05	0.1	0.15	0.3	0.45	14490	20.5	0.081	0.1	0.1	0.34	0.46
1220	15.0	0.05	0.1	0.15	0.3	0.45	14733	46.0	0.081	0.1	0.1	0.34	0.46
1900	15.0	0.045	0.1	0.15	0.3	0.45	15020	40.0	0.063	0.08	0.14	0.22	0.56
2200	10.0	0.045	0.1	0.15	0.3	0.45	15810	68.3	0.037	0.08	0.14	0.22	0.56
2800	15.0	0.04	0.1	0.15	0.3	0.45	16058	49.7	0.038	0.08	0.14	0.22	0.56
3100	20.0	0.037	0.1	0.15	0.3	0.45	16426	55.0	0.031	0.06	0.12	0.24	0.58
3400	40.0	0.037	0.1	0.15	0.3	0.45	16864	51.2	0.05	0.06	0.12	0.24	0.58
3900	20.0	0.035	0.1	0.15	0.3	0.45	17049	56.8	0.08	0.06	0.12	0.24	0.58
4490	30.0	0.035	0.1	0.15	0.3	0.45	17391	32.0	0.087	0.06	0.12	0.24	0.58
4702	41.3	0.032	0.1	0.15	0.3	0.45	17744	31.0	0.086	0.06	0.12	0.24	0.58
4822	65.0	0.032	0.1	0.15	0.3	0.45	17942	32.0	0.09	0.06	0.12	0.24	0.58
4970	98.0	0.032	0.1	0.15	0.3	0.45	18930	10.0	0.098	0.06	0.12	0.24	0.58
5068	98.0	0.032	0.1	0.15	0.3	0.45	19200	19.0	0.102	0.06	0.12	0.24	0.58
5422	98.0	0.016	0.1	0.15	0.3	0.45	19430	19.2	0.086	0.06	0.12	0.24	0.58
5536	35.0	0.016	0.1	0.15	0.3	0.45	19915	22.0	0.05	0.06	0.12	0.24	0.58
6050	12.0	0.095	0.1	0.15	0.3	0.45	20096	14.9	0.032	0.06	0.12	0.24	0.58
7940	12.0	0.095	0.1	0.15	0.3	0.45	20391	17.9	0.043	0.08	0.13	0.31	0.48
8355	15.0	0.12	0.1	0.15	0.3	0.45	21317	46.4	0.043	0.08	0.13	0.31	0.48
9365	15.0	0.12	0.1	0.15	0.3	0.45	21486	48.0	0.035	0.08	0.13	0.31	0.48
9935	25.0	0.066	0.1	0.15	0.3	0.45	21671	48.4	0.027	0.08	0.13	0.31	0.48
10400	50.0	0.036	0.1	0.1	0.34	0.46	21984	33.0	0.018	0.1	0.15	0.38	0.37
11000	51.0	0.048	0.1	0.1	0.34	0.46	22180	28.7	0.015	0.1	0.15	0.38	0.37
11250	30.0	0.048	0.1	0.1	0.34	0.46	22438	29.6	0.013	0.1	0.15	0.38	0.37
12139	37.0	0.056	0.1	0.1	0.34	0.46	22696	37.9	0.013	0.1	0.15	0.38	0.37
12318	48.0	0.064	0.1	0.1	0.34	0.46	22792	37.0	0.013	0.1	0.15	0.38	0.37
12484	38.0	0.066	0.1	0.1	0.34	0.46	23107	109.0	0.013	0.1	0.15	0.38	0.37
12689	61.8	0.057	0.1	0.1	0.34	0.46	23368	106.0	0.009	0.1	0.15	0.38	0.37
13088	45.6	0.043	0.1	0.1	0.34	0.46	23750	81.0	0.006	0.1	0.15	0.38	0.37
13516	50.5	0.035	0.1	0.1	0.34	0.46	24000	81.0	0.006	0.1	0.15	0.38	0.37
14115	69.8	0.036	0.1	0.1	0.34	0.46							

β_i (sediment composition i-th fraction), i=1,2,3,4; d_i=0.3 mm, 3 mm, 30 mm and 300 mm respectively.